## 2018 James S. Rickards Fall Invitational

For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

- 1. Which of the following is one-dimensional?
  - (A) Point (B) Solid (C) Line (D) Plane (E) NOTA
- 2. If  $\triangle ABC$  has BC = 5,  $\angle A = 30^{\circ}$ , and  $\angle C = 45^{\circ}$ , what is its area?

(A) 
$$\frac{50\sqrt{3}+75}{12}$$
 (B)  $\frac{25\sqrt{3}+75}{3}$  (C)  $\frac{25\sqrt{3}+150}{12}$  (D)  $\frac{50\sqrt{3}+75}{6}$  (E) NOTA

3. Sagar is distraught because Shravan called him a walnut, and Sagar absolutely despises walnuts. Sagar wants to measure how much of a walnut he really is, and to do so, he must find the measure of  $\angle BAD$ , given that  $\overline{AB}$  and  $\overline{AD}$  are both tangent to circle O, as shown. What is the measure of  $\angle BAD$ ?



- (A)  $80^{\circ}$  (B)  $105^{\circ}$  (C)  $100^{\circ}$  (D)  $90^{\circ}$  (E) NOTA
- 4. A certain compass can expand to a 150° angle. If the length of both legs of the compass is 15 centimeters, then what is the circumference of the biggest circle you could draw with it, in centimeters?

(A) 
$$\pi(30\sqrt{2} + 30\sqrt{6})$$
 (B)  $\pi(15\sqrt{2} + 30\sqrt{6})$  (C)  $\pi(15\sqrt{2} + 15\sqrt{6})$  (D)  $\pi(30\sqrt{2} + 15\sqrt{6})$  (E) NOTA

5. Sina wants to find the measure of angle A. If he finds that  $sin(A) = \frac{\sqrt{3}}{2}$ , what is the value of angle A in radians, given that  $0 \le A \le \frac{\pi}{2}$ ? (A)  $\frac{\pi}{2}$  (B)  $\frac{\pi}{3}$  (C)  $\frac{2\pi}{3}$  (D)  $\pi$  (E) NOTA

- 6. Jeromy's home is at the point (4,5) on the Cartesian plane. One day, he decides he wants to make an enchilada. First, he will go to the Red River (x-axis) to collect hot sauce, and then to the Tortilla Tree located at the point (8,7) for tortillas. Then, Jeromy will go to the Cheesy Canyon (y-axis) to retrieve queso, and then to the Jala Jungle at the point (5,14) for veggies. What is the shortest distance Jeromy must traverse to make his enchilada? (A) 32 (B)  $4\sqrt{10} + \sqrt{218}$  (C)  $5\sqrt{10} + 2\sqrt{109}$  (D) 64 (E) NOTA
- 7. Consider the following true statements:

If Jackson has a small head, then he has long legs. If Jackson is tall, then he has long legs.

Which of the following must also be true?

- (A) If Jackson is not tall, then he does not have long legs.
- (B) If Jackson has long legs, then Jackson has a small head.
- (C) If Jackson has a small head, then Jackson is tall.
- (D) If Jackson is short, then he does not have a small head.
- (E) NOTA

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- 8. Anurag loves history and is trying to learn everything he can about the Columbian Exchange. He was looking at a globe with a radius of 50 inches and wanted to find out how far a point on the Americas was from a point on West Europe. If the shortest distance between the two points through the interior of the globe is 40 inches, then what is the shortest distance between the two points when traveling on the surface of the globe, in inches (round your answer to the nearest tenth and assume  $\arcsin(0.4) = 23.6^{\circ}$ )?
  - (A) 41.2 (B) 20.6 (C) 25 (D) 14.1 (E) NOTA
- 9. Vishnav has become paranoid because his fans keep following him everywhere he goes! To be free of the paparazzi, Vishnav decides to construct a giant box to trap all of his fans inside; he designs it to be 50 m  $\times$  20 m  $\times$  30 m. If his fans can be represented by spheres of radius 2 m, then what is the largest number of fans Vishnav can trap in his box?
  - (A) 420 (B) 490 (C) 385 (D) 455 (E) NOTA
- 10. Varun was trying to solve a math problem, when, suddenly, he forgot how parallel lines are defined! Help Varun escape the ridicule of his peers by finding out which of the following are always true in three-dimensional space:
  - I. Two lines that never intersect are parallel
  - II. Two lines that are parallel have the same slope.
  - III. If line B is perpendicular to lines A and C, then lines A and C are parallel.
  - (A) I. and II. (B) II. (C) II. and III. (D) I., II., and III. (E) NOTA
- 11. Erin is a big fan of K-pop, and wanted to make a three-dimensional model of the letter K. To do so, she vertically cut a cone in half, and then attached the apex of each half together as shown below. What is the surface area of her model, in cm<sup>2</sup>, if the uncut cone had a radius of 4 cm and a height of 3 cm?



- (A)  $24 + 24\pi$  (B)  $12 + 24\pi$  (C)  $24 + 36\pi$  (D)  $12 + 36\pi$  (E) NOTA
- 12. Erin also wants to be a doctor when she grows up. One of the harder questions she came across was: "If an uncut cone with a radius of 4 cm and a height of 3 cm was hollowed out and filled with  $\frac{32}{3}\pi$  cm<sup>3</sup> of fluid, how high would the fluid rise, assuming the apex of the cone is pointed downwards?" What is the answer to the question?
  - (A)  $\sqrt[3]{6}$  (B)  $\sqrt[3]{18}$  (C)  $3 \sqrt[3]{6}$  (D)  $3 \sqrt[3]{18}$  (E) NOTA

(A)  $100\sqrt{5}$ 

13. Cyclic quadrilateral ABCD is shown below. If AB = 7, AD = 20, BC = 24, CD = 15, and  $BD = \frac{117}{5}$  what is the area of ABCD?



- (B) 234 (C) 246 (D)  $105\sqrt{10}$  (E) NOTA
- 14. What is the area of a polygon with an apothem of 24 and a radius of 25, given it has the same number of sides as a regular polygon with an exterior angle measure of  $0.36^{\circ}$ ?
  - (A) 168000 (B) 300000 (C) 125000 (D) 140000 (E) NOTA
- 15. Aniketh is in the mood to make himself a snow cone. First, of course, he needs some ice, but all he can find are 10 cm long cubes, which are obviously not ideal for snow cone production. Aniketh decides to cut the cubes into smaller, 5 mm long cubes. How many times greater will the surface area of a single cube of the original size be than that of a single cube of the smaller size?
  - (A) 5 (B) 40 (C) 100 (D) 400 (E) NOTA
- 16. Vishal dreams of being a famous pianist one day, and he is currently hard at work trying to master the C major scale. However, he has one obstacle that is preventing him from accomplishing his goal, which is that his brother stole his piano! He is trying to figure out where his brother could have possibly hidden the piano by calculating how much space it takes up. If Vishal's piano is in the shape of the platonic solid that has six vertices, with an edge length of 6 feet, then what is its volume, in ft<sup>3</sup>?
  - (A)  $36\sqrt{2}$  (B)  $72\sqrt{2}$  (C) 216 (D) 72 (E) NOTA
- 17. Given  $\sin(\theta) = 0.25$ , evaluate  $\cos(\theta)$  to the nearest hundredth, assuming it is positive. (A) 0.75 (B) 0.79 (C) 0.97 (D) 0.49 (E) NOTA
- 18. Consider  $\triangle ABC$ , which is inscribed in circle O, with sides AB = 6, BC = 5, and AC = 7. What is the side length of equilateral  $\triangle EFG$ , if it is circumscribed about circle O?
  - (A)  $\frac{70\sqrt{2}}{3}$  (B)  $\frac{35\sqrt{2}}{2}$  (C)  $\frac{35\sqrt{2}}{4}$  (D)  $\frac{105\sqrt{2}}{2}$  (E) NOTA

19. As Vamsi was daydreaming about the Flash, he calculated that the Flash could run in a circle of radius 10 km in just 1 second! How long, in seconds, would it take the Flash to run the entire perimeter of a regular octagon with the same area as the aforementioned circle, assuming he runs at a constant rate?

(A) 
$$\frac{4\sqrt{2\pi\sqrt{2}-2\pi}}{\pi}$$
 (B)  $\frac{2\sqrt{3\pi\sqrt{2}-2\pi}}{\pi}$  (C)  $\frac{2\sqrt{2\pi\sqrt{2}-2\pi}}{3\pi}$  (D)  $\frac{2\sqrt{2\pi\sqrt{2}-2\pi}}{\pi}$  (E) NOTA

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- 20. Aneesha and Sabrina wanted to find out who could draw a better circle, so they decided to have a contest. Sabrina ended up drawing a regular hexagon, while Aneesha ended up drawing a regular pentagon. How many more diagonals did Sabrina's "circle" have than Aneesha's "circle"?
  - (A) 3 (B) 5 (C) 2 (D) 4 (E) NOTA
- 21. Sai enjoys consuming papayas. He loves them so much that he wants to make a box out of papayas to store his papayas. He has a large solid papaya in the shape of an ellipsoid, with radii of  $\frac{3}{\pi}$ , 4, and 5 inches, which he wants to turn into a hollow cubical box that has sides 1 inch thick and has the same volume as the ellipsoid. What will be the edge length of the box, in inches?
  - (A)  $1 + \sqrt{13}$  (B)  $1 \sqrt{13}$  (C)  $2 + \sqrt{26}$  (D)  $2 + \sqrt{10}$  (E) NOTA
- 22. Given a set such that every element is a distinct positive integer, order, from least to greatest, the geometric mean, the harmonic mean, and arithmetic mean of the set.
  - (A) Geometric, Arithmetic, Harmonic
  - (B) Arithmetic, Geometric, Harmonic
  - (C) Harmonic, Geometric, Arithmetic
  - (D) Geometric, Harmonic, Arithmetic
  - (E) NOTA
- 23.  $\triangle ABC$  and convex quadrilateral BCDE share side BC, with points E and D outside of  $\triangle ABC$ . Given that AB = 4, AC = 2, BE = 1, and CD = 7, how many possible integer lengths are there for side ED?
  - (A) 8 (B) 10 (C) 11 (D) 13 (E) NOTA
- 24. Hitesh has decided to make a living by selling Butterbeer, as he is one of the rare individuals who knows the secret recipe. To protect the recipe, he hides it inside a safe with a 5-character passcode. If each character is an integer from 0-9, with consecutive characters not equal to each other, then how many possible passwords are there that could unlock the safe?
  - (A) 59049 (B) 81000 (C) 72900 (D) 65610 (E) NOTA
- 25. Two points, A(2,5) and B(4,15), lie on the Cartesian plane. Which of the following points would make triangle ABC degenerate?
  - (A) (10, 56) (B) (4, 30) (C) (5, 70) (D) (9, 65) (E) NOTA
- 26. Circle O is circumscribed around regular octagon ABCDEFGH. If the octagon has an area of  $128 + 128\sqrt{2}$ , then what is the area of circle O?

(A)  $64\pi + 64\pi\sqrt{2}$  (B)  $64\pi + 32\pi\sqrt{2}$  (C)  $32\pi + 32\pi\sqrt{2}$  (D)  $128\pi + 64\pi\sqrt{2}$  (E) NOTA

27. Varun is a huge Star Wars fan and was watching old Star Wars movies instead of learning about parallel lines. As he is an intellectual, he notices that Luke Skywalker's lightsaber is in the shape of a right cylinder of radius 2 cm, and height 50 cm, with a hemisphere of radius 2 cm on one end. What is the surface area of Luke Skywalker's lightsaber?

(A)  $216\pi \text{ cm}^2$  (B)  $220\pi \text{ cm}^2$  (C)  $224\pi \text{ cm}^2$  (D)  $212\pi \text{ cm}^2$  (E) NOTA

- 28. Joshua needs to find the area of his favorite piece of paper. He knows the paper is defined by the points (4, 5), (-2, 8), (3, -4), (8, 0), and (-5, -1). Assuming the piece of paper is convex, what is its area?
  - (A)  $\frac{179}{2}$  (B)  $\frac{9}{2}$  (C)  $\frac{95}{2}$  (D)  $\frac{85}{2}$  (E) NOTA

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29. Which of the following points does Euler's Line **NOT** pass through?

(A) Centroid (B) Incenter (C) Orthocenter (D) Circumcenter (E) NOTA

- 30. Akash is very excited about the new Fantastic Beasts movie, and he can't stop fantasizing about Dumbledore's wand, so he decided to graph it on the Cartesian plane. The wand can be represented by drawing a line that passes through points  $(\frac{27}{2}, \frac{31}{2})$  and  $(-\frac{36}{7}, -\frac{22}{7})$  and a circle centered at (3,3) with a radius of 10. What is the distance between the two points where the line intersects the circle?
  - (A)  $\sqrt{5}$  (B)  $6\sqrt{2}$  (C)  $14\sqrt{2}$  (D)  $12\sqrt{2}$  (E) NOTA